

plan strain

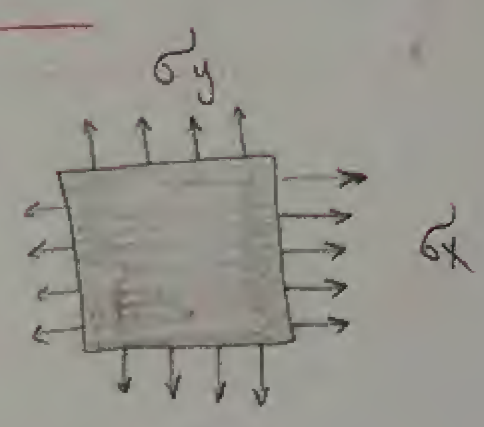
$$E = \frac{\sigma_x}{\epsilon_x}$$

$$n = \frac{\text{lateral strain}}{\text{long. strain}}$$

$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\epsilon_y = \epsilon_z = -n \cdot \epsilon_x$$

a) Given σ_x & σ_y
 Find ϵ_x & ϵ_y



$$\epsilon_x = \frac{\sigma_x}{E} - n \frac{\sigma_y}{E} \rightarrow [1]$$

$$\epsilon_y = \frac{\sigma_y}{E} - n \frac{\sigma_x}{E} \rightarrow [2]$$

b) Given ϵ_x & ϵ_y
 Find σ_x & σ_y

$$\sigma_x = \frac{E}{(1-n^2)} (\epsilon_x + n \epsilon_y) \rightarrow [3]$$

$$\sigma_y = \frac{E}{(1-n^2)} (\epsilon_y + n \epsilon_x) \rightarrow [4]$$

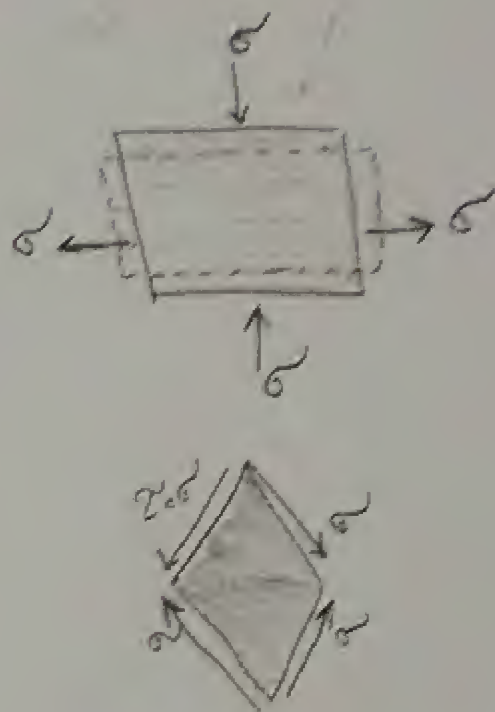
(b)

shear strain $\epsilon_s = \gamma$

$$\tau = G \cdot \gamma$$

$$\epsilon_x = \frac{\sigma}{E} + n \frac{\sigma}{E} = \frac{\sigma}{E} (1+n)$$

$$\epsilon_y = -\frac{\sigma}{E} - n \frac{\sigma}{E} = -\frac{\sigma}{E} (1+n)$$



For case (a)

$$W \cdot D = 2 * \frac{1}{2} \sigma \cdot \epsilon = \sigma \cdot \epsilon$$

For case (b)

$$\gamma = \frac{\tau}{G} = \frac{\sigma}{G}$$

$$W \cdot D = \frac{1}{2} \tau \cdot \gamma = \frac{\tau^2}{2G}$$

$$\sigma \cdot \epsilon = \frac{\tau^2}{2G}$$

$$\frac{\tau^2}{2G} = \frac{\sigma^2}{E} (1+n) = \frac{\sigma^2}{2G}$$

$$\therefore G = \frac{E}{2(1+n)}$$

Analogy between stresses & strain.

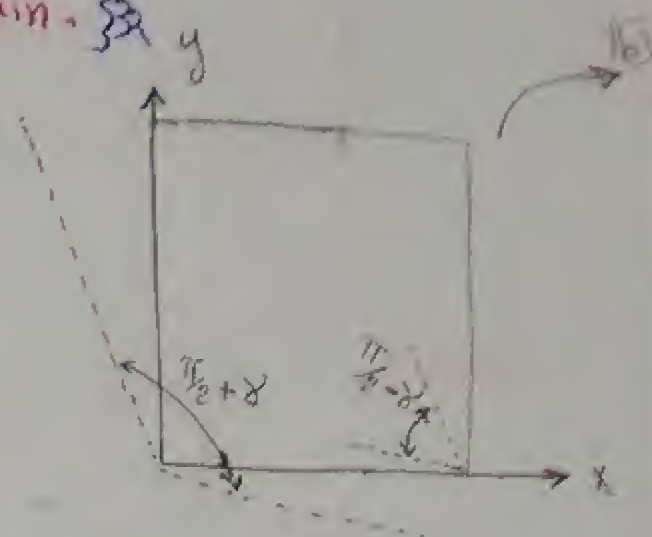
stress strain

$$\epsilon_x \rightarrow \sigma_x$$

$$\epsilon_y \rightarrow \sigma_y$$

$$\frac{1}{2} \sigma_{xy} \rightarrow \tau$$

$$\phi \rightarrow \theta$$



$$\sigma_{\theta} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

For max. stress.

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{\theta} = \frac{-(\sigma_x - \sigma_y)}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

For max. τ_{θ}

$$\tan 2\theta_s = \frac{-(\sigma_x - \sigma_y)}{2\tau}$$

$$\tau_{max} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

d

$$\text{where } \sigma_{1s} = \sigma_{2s} = \left(\frac{\sigma_x + \sigma_y}{2} \right)$$

1 shear

2 shear

strain

$$\epsilon_x = \frac{(\epsilon_x + \epsilon_y)}{2} + \frac{(\epsilon_x - \epsilon_y)}{2} \cos 2\alpha + \dots$$

$$\dots + \frac{1}{2} \gamma_{xy} \cdot \sin 2\alpha$$

$$\gamma_x = -(\epsilon_x + \epsilon_y) \sin 2\alpha + \frac{1}{2} \gamma_{xy} \cos 2\alpha$$

$$\epsilon_{1/2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \frac{1}{2} \sqrt{(\epsilon_x - \epsilon_y)^2 + \gamma_{xy}^2}$$

$$\gamma_{\max} = \frac{\epsilon_1 - \epsilon_2}{2}$$

$$\epsilon_1 = \frac{E}{(1-n^2)} (\epsilon_1 + n \epsilon_2)$$

$$\epsilon_2 = \frac{E}{(1-n^2)} (\epsilon_2 + n \epsilon_1)$$

$$\gamma_{\max} = \epsilon_1 - \epsilon_2$$

